Math 115

Fall 2017

Lecture 28

① Simplify: $\frac{4x+20}{x^2-25} = \frac{4(x+5)}{(x+5)(x-5)} = \boxed{\frac{4}{x-5}}$

② Find all excluded values: $\frac{3x+5}{2x^2+x-6}$

$2x^2+x-6=0$

$a=2 \quad b=1 \quad c=-6$

$b^2-4ac = 1^2-4(2)(-6)$

$= 49$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-1 \pm \sqrt{49}}{4} = \frac{-1 \pm 7}{4}$

$x = \frac{-1+7}{4} = \frac{6}{4} = \boxed{\frac{3}{2}}$

$x = \frac{-1-7}{4} = \frac{-8}{4} = \boxed{-2}$

E.V. -2 & $\frac{3}{2}$

③ Divide: $\dfrac{15x+5}{3x^2-14x-5} \div \dfrac{15}{3x-12}$

$= \dfrac{5(3x+1)}{(3x+1)(x-5)} \cdot \dfrac{3(x-4)}{3 \cdot 5} = \boxed{\dfrac{x-4}{x-5}}$

④ Subtract: $\dfrac{2y+7}{y^2-9} - \dfrac{y+4}{y^2-9}$

$= \dfrac{2y+7-y-4}{y^2-9} = \dfrac{y+3}{(y+3)(y-3)} = \boxed{\dfrac{1}{y-3}}$

⑤ Simplify: $\dfrac{5}{x^2+6x+5} - \dfrac{3}{x^2-25}$

$= \dfrac{5(x-5)}{(x+5)(x+1)(x-5)} - \dfrac{3(x+1)}{(x+5)(x-5)(x+1)}$

$= \dfrac{5(x-5)-3(x+1)}{(x+5)(x+1)(x-5)} = \dfrac{5x-25-3x-3}{(x+5)(x+1)(x-5)} = \boxed{\dfrac{2x-28}{\text{LCD}}}$

⑥ Solve: $\dfrac{5x}{x+2} + 3 = \dfrac{4x-6}{x+2}$

LCD $= x+2$, E.V. -2

$(x+2) \cdot \dfrac{5x}{x+2} + (x+2) \cdot 3 = (x+2) \cdot \dfrac{4x-6}{x+2}$

$5x+3(x+2) = 4x-6$

$5x+3x+6 = 4x-6$

$4x = -12$

$\boxed{x=-3}$

$\{-3\}$

7) Solve: $\frac{3}{x+5} + \frac{1}{x-5} = \frac{10}{x^2-25}$

LCD = $(x+5)(x-5)$ E.V.: ± 5

$3(x-5) + 1(x+5) = 10$

$3x - 15 + x + 5 = 10$

$4x - 10 = 10$

$4x = 20$

$x = 5$

$\varnothing$

5 is an E.V.

Solve: $\frac{5}{x+1} = \frac{3}{x-2}$

LCD = $(x+1)(x-2)$

E.V.: $-1, 2$

$5(x-2) = 3(x+1)$ Cross-multiply.

$5x - 10 = 3x + 3$

$5x - 3x = 3 + 10$

$2x = 13$

$x = 13/2$ $\{13/2\}$

Solve $x + 5 = \frac{6}{x}$ LCD = x, E.V. = 0

$x^2 + 5x = 6$

$x^2 + 5x - 6 = 0$ → $a=1$ $b=5$, $c=-6$

$(x+6)(x-1)=0$

$x=-6$ ✓ $x=1$ ✓

$\{-6, 1\}$

$b^2-4ac = 5^2-4(1)(-6)$

$= 25 + 24$

$= 49$

$x = \frac{-5 \pm \sqrt{49}}{2(1)} = \frac{-5 \pm 7}{2}$

$x = \frac{-5+7}{2} = \frac{2}{2} = 1$

$x = \frac{-5-7}{2} = \frac{-12}{2} = -6$

Solve $10x = 7 - \frac{1}{x}$ LCD = x, E.V. = 0

$10x^2 = 7x - 1$

$10x^2 - 7x + 1 = 0$

$a=10$ $b=-7$ $c=1$

$b^2-4ac = (-7)^2-4(10)(1)$

$= 9$

$x = \frac{7+3}{20} = \frac{10}{20} = \boxed{\frac{1}{2}}$ $x = \frac{7-3}{20} = \frac{4}{20} = \boxed{\frac{1}{5}}$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-7) \pm \sqrt{9}}{2(10)}$

$x = \frac{7 \pm 3}{20}$

$\{\frac{1}{2}, \frac{1}{5}\}$

Solve $2 - \frac{7}{x} = \frac{4}{x^2}$ LCD: x^2

$2x^2 - \frac{7}{x}\cdot x^2 = \frac{4}{x^2}\cdot x^2$ E.V.: 0

$2x^2 - 7x = 4$ $a=2$ $b=-7$ $c=-4$

$2x^2 - 7x - 4 = 0$ $b^2-4ac = (-7)^2 - 4(2)(-4)$

$= 49 + 32 = 81$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{7 \pm \sqrt{81}}{4} = \frac{7 \pm 9}{4}$$

$x = \frac{7+9}{4} = \frac{16}{4} = 4$

$x = \frac{7-9}{4} = \frac{-2}{4} = -\frac{1}{2}$

$\left\{-\frac{1}{2}, 4\right\}$

Basic Math

Simplify: $\dfrac{\frac{1}{2} - \frac{1}{3}}{\frac{1}{6}} = \dfrac{6\cdot\frac{1}{2} - 6\cdot\frac{1}{3}}{6\cdot\frac{1}{6}}$

LCD = 6

$= \frac{3-2}{1} = \frac{1}{1} = \boxed{1}$

Simplify $\dfrac{1 - \frac{3}{x}}{x - \frac{9}{x}} = \dfrac{x\cdot 1 - x\cdot\frac{3}{x}}{x\cdot x - x\cdot\frac{9}{x}} = \dfrac{x-3}{x^2-9}$

LCD = x

$= \dfrac{1(x-3)}{(x-3)(x+3)} = \boxed{\dfrac{1}{x+3}}$

Simplify: $\dfrac{\frac{x}{4}-\frac{4}{x}}{1-\frac{4}{x}} = \dfrac{4x\cdot\frac{x}{4}-4x\cdot\frac{4}{x}}{4x\cdot 1-4x\cdot\frac{4}{x}}$

LCD $= 4x$

$$= \frac{x^2-16}{4x-16}$$

$$= \frac{(x+4)(x-4)}{4(x-4)} = \boxed{\frac{x+4}{4}}$$

Simplify $\dfrac{1+\frac{4}{x}+\frac{4}{x^2}}{1-\frac{4}{x^2}}$

LCD $= x^2$

$$= \frac{x^2+4x+4}{x^2-4}$$

$$= \frac{(x+2)(x+2)}{(x+2)(x-2)} = \boxed{\frac{x+2}{x-2}}$$

Simplify: $\dfrac{5x^{-1}-2y^{-1}}{25x^{-2}-4y^{-2}} = \dfrac{\frac{5}{x}-\frac{2}{y}}{\frac{25}{x^2}-\frac{4}{y^2}}$

LCD $= x^2y^2$

$$\frac{x^2y^2\cdot\frac{5}{x}-x^2y^2\cdot\frac{2}{y}}{x^2y^2\cdot\frac{25}{x^2}-x^2y^2\cdot\frac{4}{y^2}} = \frac{5xy^2-2x^2y}{25y^2-4x^2}$$

$$= \frac{xy(5y-2x)}{(5y-2x)(5y+2x)} = \boxed{\frac{xy}{5y+2x}}$$

$(5y)^2-(2x)^2$

Unfinished Problem from yesterday

$$\frac{1}{x}+\frac{1}{x+1}=\frac{3}{2}$$

LCD $= 2x(x+1)$

E.V.: $0, -1$

$$2x(x+1)\cdot\frac{1}{x}+2x(x+1)\cdot\frac{1}{x+1}=2x(x+1)\cdot\frac{3}{2}$$

$$2(x+1)+2x = 3x(x+1)$$

$$2x+2+2x = 3x^2+3x$$

$$4x+2 = 3x^2+3x$$

$$3x^2+3x-4x-2=0$$

$$3x^2-x-2=0$$

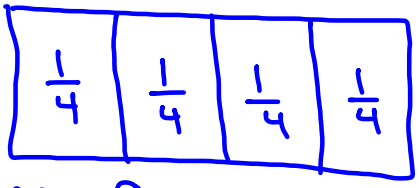
$$(3x+2)(x-1)=0$$

$x \neq -\frac{2}{3}$ $\quad$ $x=1$

This problem was for 2 Cons. integers x, $x+1$

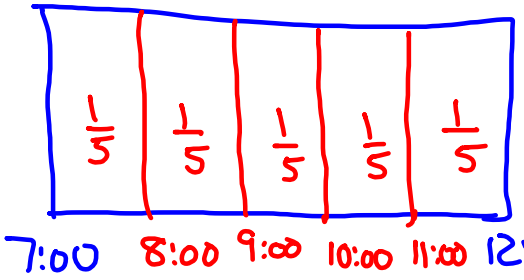
$-\frac{2}{3}$ is not an integer. 1 & 2

Brian can paint this room in 4 hrs.



His rate $\frac{1}{4}$ of the job / hr.

I can paint the same room in 5 hrs,



My rate is $\frac{1}{5}$ / hr.

How long does it take to paint the room working together?

work = How long · How fast

= time · Rate

$$\underbrace{\frac{1}{4} \cdot t}_{\text{work by Brian}} + \underbrace{\frac{1}{5} \cdot t}_{\text{work by me}} = \underbrace{1}_{\text{complete work}}$$

$\frac{t}{4} + \frac{t}{5} = 1$

LCD = 20

$5t + 4t = 20$

$9t = 20$

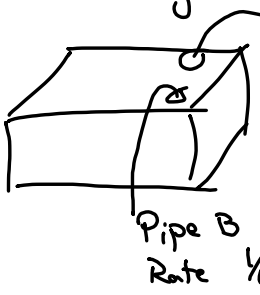
$t = \frac{20}{9}$

$t \approx 2.2$ hrs

Pipe A can fill up a pool in 6 hrs.

Pipe B - - - - - - - - 10 hrs.

How long if they work together?



$\frac{1}{6}\cdot t + \frac{1}{10}\cdot t = 1$ (Pipe A, Pipe B, 1 comp. work)

$\frac{t}{6} + \frac{t}{10} = 1$

LCD = 30

$5t + 3t = 30$

$8t = 30$

$t = 3.75$

3 hrs 45 mins.

3.75 hrs

Faucet can fill up the sink in 3 mins.

Drain can empty the sink in 5 mins.

If faucet is running, and drain is left open, How long does it take to fill up the sink?

$\frac{1}{3}\cdot t - \frac{1}{5}\cdot t = 1$ comp. job

(faucet, drain)

Solve $\frac{t}{3} - \frac{t}{5} = 1$

LCD = 15

$5t - 3t = 15$

$2t = 15$

$t = 7.5$

7.5 minutes.

It takes Ana 18 minutes longer than Joe to do a job.

Together, they can do it in 12 minutes.

How long if they work alone?

		Rate
Joe	x	$\frac{1}{x}$
Ana	$x+18$	$\frac{1}{x+18}$

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$\frac{1}{x} \cdot 12 + \frac{1}{x+18} \cdot 12 = 1$

Joe Ana

Solve

$\frac{12}{x} + \frac{12}{x+18} = 1$

$LCD = x(x+18)$

E.V. : $0, -18$

$12(x+18) + 12x = x(x+18)$

$12x + 216 + 12x = x^2 + 18x$

$x^2 + 18x - 24x - 216 = 0$

$x^2 - 6x - 216 = 0$

$(x+12)(x-18) = 0$

$x = -12$ ~~$x = -12$~~ $x = 18$

Joe 18 minutes

Ana 36 minutes

It takes Wilson 3 times longer than Roger to do a job.

Together, they can do it in 2 hrs.

How long if they work alone?

		Rate
Roger	x	$\frac{1}{x}$
Wilson	$3x$	$\frac{1}{3x}$

$\frac{1}{x} \cdot 2 + \frac{1}{3x} \cdot 2 = 1$

Roger Wilson

Solve $\frac{2}{x} + \frac{2}{3x} = 1$

$LCD = 3x$

$6 + 2 = 3x$

$8 = 3x$

$x = \frac{8}{3}$

Roger $\frac{8}{3}$ hrs, Wilson 8 hrs

Ashley takes 3 times longer than taylor.

Together → 21 Days.

How long alone?

Taylor → x → $\frac{1}{x}$

Ashley → $3x$ → $\frac{1}{3x}$

$\frac{1}{x} \cdot 21 + \frac{1}{3x} \cdot 21 = 1$

Solve

$\frac{21}{x} + \frac{\cancel{21}^{7}}{\cancel{3x}} = 1$

$\frac{21}{x} + \frac{7}{x} = 1$

LCD $= x$

$21 + 7 = x$

$28 = x$

Taylor 28 Days

Ashley $3(28) = 84$ Days